

# Richards Middle School

## 2024 Summer Math Packet

### for Rising Enhanced Algebra Students

**Directions:**

**You MUST SHOW ALL WORK in order to receive credit.**

**No work shown will earn you a grade of zero.**

Neatly show your work and circle your answers. If there is not enough room to show your work, use notebook paper front and back. Neatly organize and number your problems. Circle your answers!

**Due Date:**

**Monday, August 12, 2024**

No late work will be accepted. You can turn in your Math packet early. Start off your 8<sup>th</sup> grade year right... get this done and turned in on time!

**FAQ:**

**Will these be graded?**

Yes, these packets will be graded. Do your best. In addition, you will be tested over all the material on the Summer Math Packet.

**What if I don't know how to something?**

Read through the hints/guides/examples on each page before you do the problems. You may also get help from others if you need it.

**What do you mean by "help"?**

Help means help, not copying answers from a friend, website or app.

**Why do we have to do a Summer math packet?**

Because it is an excellent review of important math concepts, and great preparation for Enhanced Algebra.

**What if I don't do this packet?**

You will get a zero. Zeros are bad. Do your packet.

**Fraction Operations**

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8} = \frac{1+6}{8} = \frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if it was not so apparent? For example:  $\frac{7}{12} + \frac{8}{15} =$

For this example, we must find the Lowest Common Denominator (LCM) for the two denominators 12 and 15.

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, ...

Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, ...

The LCM of 12 and 15 is 60

So,  $\frac{7}{12} + \frac{8}{15} = \frac{35}{60} + \frac{32}{60} = \frac{35+32}{60} = \frac{67}{60} = 1\frac{7}{60}$ .

Note: Be sure that answers are always in lowest terms

To multiply fractions, we multiply the numerators together and denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \quad \text{and} \quad \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$$

Exercises: Perform the indicated operation

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1.  $\frac{6}{7} + \frac{2}{3} =$

2.  $\frac{8}{9} + \frac{3}{4} =$

3.  $\frac{9}{11} - \frac{2}{5} =$

4.  $\frac{5}{7} - \frac{5}{9} =$

5.  $\frac{6}{11} \cdot \frac{2}{3} =$

6.  $\frac{7}{9} \cdot \frac{3}{5} =$

7.  $\frac{6}{7} \div \frac{1}{5} =$

8.  $\frac{7}{11} \div \frac{3}{5} =$

9.  $\left[ \frac{2}{3} - \frac{5}{9} \right] \div \left[ \frac{4}{7} + \frac{1}{6} \right] =$

10.  $\frac{3}{4} + \frac{4}{5} \left[ \frac{5}{9} + \frac{9}{11} \right] =$

11.  $\left[ \frac{3}{4} + \frac{4}{5} \right] \left[ \frac{5}{9} + \frac{9}{11} \right] =$

**Rename Fractions, Percents, and Decimals**

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as  $\frac{3}{5}$ , and divide the numerator (the top number of the fraction) by the denominator (the bottom number of the fraction). So:

$$5 \overline{)3.0} \quad \text{and the fraction } \frac{3}{5} \text{ is equivalent to the decimal } 0.6$$

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a number by 100), so  $0.6 \cdot 100 = 60$  and  $\frac{3}{5} = 0.6 = 60\%$ .

To convert a percent to a decimal, we divide the percent by 100,  $60\%$  is the same as  $60 \div 100$ , which is 0.6, so  $60\% = 0.6$

To convert a fraction into a percent, we can use proportions to solve, so

$$\frac{3}{5} = \frac{x}{100} \text{ and using cross products to solve, } 5x = 300 \text{ or } x = 60\%$$

Exercises: Complete the chart

	Fraction	Decimal	Percent
1.		0.04	
2.			125%
3.	$\frac{2}{3}$		
4.		1.7	
5.			0.6%
6.	$3\frac{1}{2}$		
7.		0.9	
8.			70%
9.	$\frac{17}{25}$		
10.		0.007	

**Add and Subtract Mixed Numbers**

Hints/Guide:

When adding mixed numbers, we can add the whole numbers and the fractions separately, then simplify the answer. For example:

$$4\frac{1}{3} + 2\frac{3}{4} = 4\frac{8}{24} + 2\frac{18}{24} = 6\frac{26}{24} = 6 + 1\frac{2}{24} = 7\frac{2}{24} = 7\frac{1}{12}$$

When subtracting mixed numbers, we subtract the whole numbers and the fractions separately, then simplify the answer. For example:

$$7\frac{3}{4} - 2\frac{15}{24} = 7\frac{18}{24} - 2\frac{15}{24} = 5\frac{3}{24} = 5\frac{1}{8}$$

$$5\frac{1}{4} - 3\frac{3}{8} = 5\frac{2}{8} - 3\frac{3}{8} = 4\frac{10}{8} - 3\frac{3}{8} = 1\frac{5}{8}$$

Note: regrouping needed in order to subtract

Exercises: Solve in lowest terms.

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1.  $3\frac{1}{2} + 5\frac{3}{5} =$

2.  $6\frac{17}{25} + 8\frac{4}{7} =$

3.  $6\frac{2}{3} + 9\frac{7}{9} =$

4.  $8\frac{3}{10} - 6\frac{7}{9} =$

5.  $9\frac{7}{15} - 2\frac{7}{12} =$

6.  $12\frac{8}{9} - 7\frac{3}{4} =$

**Multiply and Divide Mixed Numbers**

Hints/Guide:

To multiply mixed numbers, we can first convert the mixed numbers into improper fractions. This is done by multiplying the denominator by the whole number part of the mixed number and then adding the numerator to this product. This sum is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number.

For example:  $3\frac{2}{5}$  leads to  $3 \cdot 5 + 2 = 17$ , so  $3\frac{2}{5} = \frac{17}{5}$ .

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example:  $5\frac{1}{5} \cdot 3\frac{1}{2} = \frac{26}{5} \cdot \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$

To divide mixed numbers, we must convert to improper fractions then multiply by the reciprocal of the second fraction and simplify. For example:  $2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \cdot \frac{3}{10} = \frac{15}{20} = \frac{3}{4}$

Exercises: Solve in lowest terms.

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1.  $6\frac{2}{3} \cdot 7\frac{3}{7} =$

2.  $3\frac{1}{3} \cdot 6\frac{4}{5} =$

3.  $7\frac{1}{8} \cdot 6 =$

4.  $4\frac{1}{4} \div \frac{5}{7} =$

5.  $3\frac{2}{3} \div 4\frac{3}{7} =$

6.  $\frac{3}{4} \div 2\frac{3}{11} =$

**Find Percent of a Number**

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut by moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

$$4.5\% \text{ of } 240 = 4.5\% \cdot 240 = 0.045 \cdot 240 = 10.8$$

Exercises: Solve for n.

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1.  $30\% \text{ of } 40 = n$

2.  $7.5\% \text{ of } 42 = n$

3.  $150\% \text{ of } 320 = n$

4.  $15\% \text{ of } 54 = n$

5.  $0.65\% \text{ of } 320 = n$

6.  $80\% \text{ of } 9 = n$

7.  $9\% \text{ of } 7 = n$

8.  $150\% \text{ of } 38 = n$

9.  $215\% \text{ of } 348 = n$

10.  $70\% \text{ of } 30 = n$

**Solve Problems Using Percents**

Hints/Guide:

When solving percent problems, we apply the rules for finding percent of a number in realistic situations. For example, to find the amount of sales tax on a \$450.00 item if the tax rate is 5%, we find 5% of 450 ( $.05 \cdot 450 = 22.5$ ), and then label our answer in dollars, getting \$22.50.

Exercises: Solve the following items.

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. Susie has just bought a pair of jeans for \$49.95, a sweater for \$24.50, and a jacket for \$85.95. The sales tax is 5%. What is her total bill?
2. Jack bought a set of golf clubs for \$254.00 and received a rebate of 24%. How much was the rebate?
3. A construction manager calculates it will cost \$2,894.50 for materials for her next project. She must add in 12.5% for scrap and extras. What will be the total cost?
4. The regular price for a video game system is \$164.50 but is on sale for 30% off. What is the amount of the discount?

What is the sale price?

5. Cindy earns a 15% commission on all sales. On Saturday, she sold \$985.40 worth of merchandise. What was the amount of commission she earned on Saturday?
6. The band had a fundraiser and sold \$25,800 worth of candy. They received 38% of this amount for themselves. How much did they receive?

**Integers I**

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse. For example,  $6 - 11 = 6 + -11 = -5$

Exercises: Solve the following problems.

1.  $(-4) + (-5) =$

2.  $-9 - (-2) =$

3.  $6 - (-9) =$

4.  $(-6) - 7 =$

5.  $7 - (-9) =$

6.  $15 - 24 =$

7.  $(-5) + (-8) =$

8.  $-15 + 8 - 8 =$

9.  $14 + (-4) - 8 =$

10.  $14.5 - 29 =$

11.  $-7 - 6.85 =$

12.  $-8.4 - (-19.5) =$

13.  $29 - 16 + (-5) =$

14.  $-15 + 8 - (-19.7) =$

15.  $45.6 - (-13.5) + (-14) =$

16.  $-15.98 - 6.98 - 9 =$

17.  $-7.24 + (-6.28) - 7.3 =$

18.  $29.45 - 56.009 - 78.2 =$

19.  $17.002 + (-7) - (-5.23) =$

20.  $45.9 - (-9.2) + 5 =$



**Integers II**

Hints/Guide:

The rules for multiplying integers are:

Positive · Positive = Positive

Positive · Negative = Negative

Negative · Negative = Positive

Negative · Positive = Negative

The rules for dividing integers are the same as multiplying integers

Exercises: Solve the following problems.

1.  $4 \cdot (-3) \cdot 6 =$

2.  $5(-12) \cdot (-4) =$

3.  $(4)(-2)(-3) =$

4.  $\frac{(-5)(-6)}{-2} =$

5.  $\frac{6(-4)}{8} =$

6.  $\frac{-56}{2^3} =$

7.  $6(-5 - (-6)) =$

8.  $8(-4 - 6) =$

9.  $-6(9 - 11) =$

10.  $\frac{-14}{2} + 7 =$

11.  $8 - \frac{-15}{-3} =$

12.  $-3 + \frac{-12 \cdot (-5)}{4} =$

13.  $\frac{-6 - (-8)}{-2} =$

14.  $-7 + \frac{4 + (-6)}{-2} =$

15.  $45 - 14(5 - (-3)) =$

16.  $(-4 + 7)(-16 + 3) =$

17.  $16 - (-13)(-7 + 5) =$

18.  $\frac{4 + (-6) - 5 - 3}{-6 + 4} =$

### Inequalities



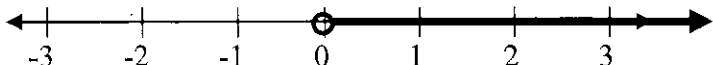
**Hints/Guide:**

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality has many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

Inequality - a statement containing one of the following symbols:

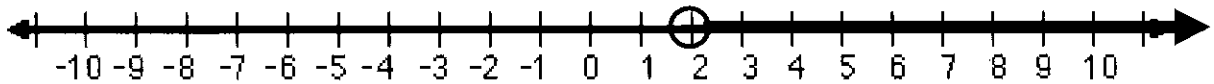
- $<$  is less than
- $>$  is greater than
- $\leq$  is less than or equal to
- $\geq$  is greater than or equal to
- $\neq$  is not equal to

**Examples:**

1. Integers between -4 and 4. 
2. All numbers between -4 and 4. 
3. The positive numbers. 

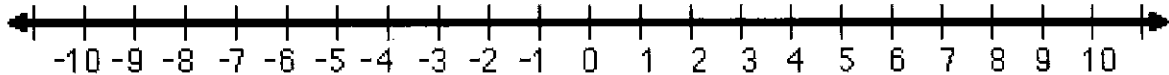
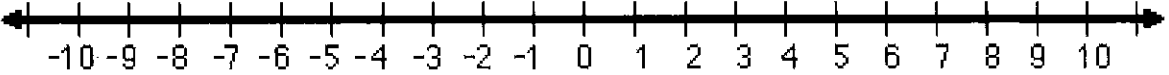
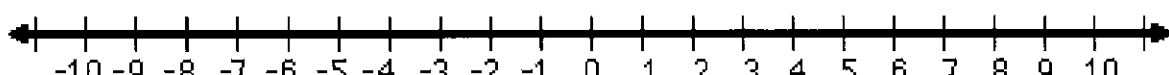
So, to solve the inequality  $-4x < -8$  becomes  $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore  $x > 2$  is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:



**Exercises:** Solve the following problems:

No Calculators!

1.  $4x > 9$   

2.  $-5t \geq -15$   

3.  $\frac{x}{2} \geq 3$   

4.  $\frac{x}{-4} > 2$

# 8<sup>th</sup> Grade – Summer Math Packet

## Domain: EXPRESSIONS & EQUATIONS

### CCRS Standards:

10 – Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

10a – Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem.

**Objective:** Write and solve linear inequalities.

**Example:** One-half of a number,  $x$ , minus 3 is at least 4. What are the possible values of the number?

Write the inequality:

One-half • a number  $- 3 \leq 4$       Reason the problem with the correct operations.

$\frac{1}{2} \cdot n - 3 \leq 4$       Replace items with variables and values

$2 \left( \frac{1}{2}n - 3 \right) \leq 2(4)$       Multiply both sides by 2

$2 \cdot \frac{1}{2}n - 2 \cdot 3 \leq 8$       distribute 2 through parentheses; multiply 2 and 4

$n - 6 \leq 8$       Simplify

$\quad +6 \quad +6$       add 6 to both sides

$n + 0 \leq 14$       Solution

$n \leq 14$       Simplify

1. Write the inequality and solve.

A certain minivan has a maximum carrying capacity of 1100 pounds. If the luggage weights 120 pounds, what is the maximum weight,  $w$ , allowable for passengers?

2. Write the inequality and solve.

Nicole swims 40 meters per minute, and she wants to swim at least 2000 meters this morning. How many minutes,  $t$ , should she swim?

3. Write the inequality and solve.

The sum of twice a number,  $n$ , and 5 is at most 15. What are the possible values for the number?

4. Write the inequality and solve.

Two-thirds of a number,  $n$ , plus 5 is greater than 12. What are the possible values for the number?

# 8<sup>th</sup> Grade – Summer Math Packet

**Domain: EXPRESSIONS & EQUATIONS**

**CCRS Standards:**

10 – Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**Objective:** Write an algebraic expression to represent unknown quantities.

The tables below show phrases written as mathematical expressions.

Phrases	Expression
12 more than a number, $x$ the sum of 12 and a number, $x$ a number, $x$ , plus 12 a number, $x$ , increased by 12 the total of $x$ and 12	$x + 12$
Phrases	Expression
3 multiplied by $g$ 3 times a number, $g$ the product of $g$ and 3	$3g$

Phrases	Expression
2 less than a number, $n$ a number, $n$ , minus 2 2 subtracted from number, $n$ a number, $n$ , decreased by 4 the difference of $n$ and 2	$n - 2$
Phrases	Expression
a number divided by 5 the quotient of $m$ and 5 divide a number by 5	$\frac{m}{5}$

<p>1.) 18 less than <math>p</math></p>	<p>2.) the quotient of a number, <math>n</math>, and 9</p>
<p>3.) 18 years older than Jordan</p>	<p>4.) 5 times as many hits as Paul</p>
<p>5.) Let <math>t</math> = the number of tomatoes Tara planted last year. This year, she planted 3 times as many. Write an algebraic expression to show how many tomatoes Tara planted this year.</p>	<p>6.) Last week, Jack sold <math>x</math> number of hot dogs at the football game. This week he sold twice as many as last week, and then he sold 10 more. Write an expression to show how many hot dogs Jack sold this week.</p>

# Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
  - Parentheses/Brackets (inside to outside)
  - Exponents
  - Multiplication/Division (left to right)
  - Addition/Subtraction (left to right)

ex:  $9x^2 - 4(y + 3z)$   
for  $x = -3, y = 2, z = 5$

$$9(-3)^2 - 4(2 + 3 \cdot 5)$$

$$9(-3)^2 - 4(2 + 15)$$

$$9(-3)^2 - 4 \cdot 17$$

$$9 \cdot 9 - 4 \cdot 17$$

$$81 - 4 \cdot 17$$

$$81 - 68 = \boxed{13}$$

# The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex:  $5(8x - 3)$

$$5(8x - 3)$$

$$5(8x) - 5(3)$$

$$\boxed{40x - 15}$$

# Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex:  $2(3x - 4) - 12x + 9$

$$2(3x - 4) - 12x + 9$$

$$6x - 8 - 12x + 9$$

$$\boxed{-6x + 1}$$

Evaluate each expression for  $a = 9$ ,  $b = -3$ ,  $c = -2$ ,  $d = 7$ . Show your work.

1. $a - cd$	2. $2b^3 + c^2$	3. $\frac{a + d - c}{b}$	4. $(a - b)^2 + d(a + c)$
5. $4c - (b - a)$	6. $\frac{a}{b} - 5a$	7. $2bc + d(12 - 5)$	8. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$	10. $7(y + 3)$	11. $-3(4w - 3)$	12. $(6r + 3)2$
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Simplify each expression, showing all work.

13. $8(x + 1) - 12x$	14. $6w - 7 + 12w - 3z$	15. $9n - 8 + 3(2n - 11)$	16. $3(7x + 4y) - 2(2x + y)$
17. $(15 + 8d)(-5) - 24d + d$	18. $9(b - 1) - c + 3b + c$	19. $20f - 4(5f + 4) + 16$	20. $8(h - 4) - h - (h + 7)$

## Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex:  $-18 = 6j$

$$\frac{-18}{6} = \frac{6j}{6}$$

$$-3 = j \rightarrow \boxed{j = -3}$$

## Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex:  $\frac{a}{7} - 12 = -9$

$$\frac{a}{7} - 12 = -9$$

$$+ 12 \quad + 12$$

$$\frac{a}{7} = 3 \times 7$$

$$\boxed{a = 21}$$

## Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex:  $5(2x - 1) = 3x + 4x - 1$

$$10x - 5 = 3x + 4x - 1$$

$$10x - 5 = 7x - 1$$

$$- 7x \quad - 7x$$

$$3x - 5 = -1$$

$$+ 5 \quad + 5$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

Solve each equation, showing all work.

21.  $f - 64 = -23$

22.  $-7 = 2d$

23.  $\frac{b}{-12} = -6$

24.  $13 = m + 21$

25.  $5x - 3 = -28$

26.  $\frac{w + 8}{-3} = -9$

27.  $-8 + \frac{h}{4} = 13$

28.  $22 = 6y + 7$

29.  $8x - 4 = 3x + 1$

30.  $-2(5d - 8) = 20$

31.  $7r + 21 = 49r$

32.  $-9g - 3 = -3(3g + 2)$

33.  $5(3x - 2) = 5(4x + 1)$

34.  $3d - 4 + d = 8d - (-12)$

35.  $f - 6 = -2f + 3(f - 2)$

36.  $-2(y - 1) = 4y - (y + 2)$